

Übungen Gleichungen, Ungleichungen, Polynomdivision – Lösungen

Lösung von Aufgabe 1:

$$\begin{aligned}\frac{8-x}{2} - \frac{2x-11}{x-3} &= \frac{x-2}{6}, \quad x \neq 3 \\ (8-x)(x-3) - 2(2x-11) &= \frac{2}{6}(x-2)(x-3) \\ 8x - x^2 - 24 + 3x - 4x + 22 &= \frac{1}{3}(x^2 - 2x - 3x + 6) \\ -x^2 + 7x - 2 &= \frac{1}{3}x^2 - \frac{5}{3}x + 2 \\ 0 &= \frac{4}{3}x^2 - \frac{26}{3}x + 4 \\ 0 &= x^2 - \frac{13}{2}x + 3 \\ x_{1/2} &= \frac{13}{4} \pm \sqrt{\frac{169}{16} - \frac{48}{16}} = \frac{13}{4} \pm \frac{11}{4} \\ x_1 &= 6, \quad x_2 = \frac{1}{2}\end{aligned}$$

$$\begin{aligned}\frac{3}{x-2} - \frac{8}{4-3x} &= \frac{19}{2x+1}, \quad x \neq 2, x \neq \frac{4}{3}, x \neq -\frac{1}{2} \\ 3(4-3x)(2x+1) - 8(x-2)(2x+1) &= 19(x-2)(4-3x) \\ 3(8x-6x^2+4-3x) - 8(2x^2-4x+x-2) &= 19(4x-8-3x^2+6x) \\ -18x^2+15x+12-16x^2+24x+16 &= -57x^2+190x-152 \\ -34x^2+39x+28 &= -57x^2+190x-152 \\ 23x^2-151x+180 &= 0 \\ x^2 - \frac{151}{23}x + \frac{180}{23} &= 0 \\ x_{1/2} &= \frac{151}{46} \pm \sqrt{\left(\frac{151}{46}\right)^2 - \frac{180}{23}} = \frac{151}{46} \pm \frac{79}{46} \\ x_1 &= \frac{230}{46} = 5, \quad x_2 = \frac{72}{46} = \frac{36}{23}\end{aligned}$$

$$\begin{aligned}(x^2-5)^2 + (x^2-1)^2 &= 40 \\ x^4 - 10x^2 + 25 + x^4 - 2x^2 + 1 &= 40 \\ 2x^4 - 12x^2 - 14 &= 0 \\ x^4 - 6x^2 - 7 &= 0 \quad z := x^2 \\ z^2 - 6z - 7 &= 0 \\ z_{1/2} &= 3 \pm \sqrt{9+7} \\ z_1 &= 7, \quad z_2 = -1 \\ z &= x^2 \Rightarrow 7 = x^2 \Rightarrow x_{1/2} = \pm\sqrt{7}\end{aligned}$$

Lösung von Aufgabe 2:

$$9\sqrt{5x+1} = 20 + 4\sqrt{5x+1}$$

$$5\sqrt{5x+1} = 20$$

$$\sqrt{5x+1} = 4$$

$$5x+1 = 16$$

$$5x = 15$$

$$x = 3$$

$$\text{Probe: } 9\sqrt{5 \cdot 3 + 1} = 20 + 4\sqrt{5 \cdot 3 + 1}$$

$$\sqrt{x+9} + \sqrt{x-12} = \sqrt{x} + \sqrt{x-7}$$

$$x+9 + 2\sqrt{(x+9)(x-12)} + x-12 = x + 2\sqrt{x(x-7)} + x-7$$

$$4 + 2\sqrt{(x+9)(x-12)} = 2\sqrt{x(x-7)}$$

$$2 + \sqrt{(x+9)(x-12)} = \sqrt{x(x-7)}$$

$$4 + 4\sqrt{(x+9)(x-12)} + (x+9)(x-12) = x(x-7)$$

$$4 + 4\sqrt{(x+9)(x-12)} + x^2 + 9x - 12x - 108 = x^2 - 7x$$

$$4\sqrt{(x+9)(x-12)} = -4x + 104$$

$$\sqrt{(x+9)(x-12)} = -x + 26$$

$$(x+9)(x-12) = (-x+26)^2$$

$$x^2 + 9x - 12x - 108 = x^2 - 52x + 676$$

$$49x = 784$$

$$x = 16$$

$$\text{Probe: } \sqrt{16+9} + \sqrt{16-12} = \sqrt{16} + \sqrt{16-7}$$

$$\sqrt{37 - 7\sqrt{5x+4}} = 4$$

$$37 - 7\sqrt{5x+4} = 16$$

$$21 = 7\sqrt{5x+4}$$

$$3 = \sqrt{5x+4}$$

$$9 = 5x+4$$

$$5 = 5x \Rightarrow x = 1$$

$$\text{Probe: } \sqrt{37 - 7\sqrt{5 \cdot 1 + 4}} = 4$$

$$2\sqrt{x} - \sqrt{2x} = 2 + \sqrt{2}$$

$$2\sqrt{x} - \sqrt{2} \cdot \sqrt{x} = 2 + \sqrt{2}$$

$$(2 - \sqrt{2})\sqrt{x} = 2 + \sqrt{2}$$

$$\sqrt{x} = \frac{2 + \sqrt{2}}{2 - \sqrt{2}} = \frac{(2 + \sqrt{2})(2 + \sqrt{2})}{(2 - \sqrt{2})(2 + \sqrt{2})} = \frac{(2 + \sqrt{2})^2}{2} = 3 + 2\sqrt{2}$$

$$x = (3 + 2\sqrt{2})^2 = 17 + 12\sqrt{2}$$

Lösung von Aufgabe 3:

$$\begin{aligned}(x^3 - 1) : (x - 1) &= (x^3 + 0x^2 + 0x - 1) : (x - 1) = x^2 + x + 1 \\(2x^3 + 11x^2 + 12x - 9) : (x + 3) &= 2x^2 + 5x - 3 \\(3a^2 + 5ab + 2b^2) : (a + b) &= 3a + 2b\end{aligned}$$

Lösung von Aufgabe 4:

$$2x + 3y = 8 \quad (I)$$

$$3x - 6y = -30 \quad (II)$$

$$2(I) + (II) \Rightarrow 7x = -14 \Rightarrow x = -2, \quad y = \frac{1}{3}(8 - 2x) = \frac{12}{3} = 4$$

$$\frac{1}{x} + \frac{2}{y} = 3 \iff \frac{1}{x} = 3 - \frac{2}{y}$$

$$\frac{5}{x} - \frac{1}{y} = 4$$

$$5 \cdot \left(3 - \frac{2}{y}\right) - \frac{1}{y} = 4$$

$$15 - \frac{11}{y} = 4$$

$$11 = \frac{11}{y} \Rightarrow y = 1, \quad \frac{1}{x} = 3 - \frac{2}{y} = 1 \Rightarrow x = 1$$

$$3x + 2y = 9$$

$$xy = 3 \Rightarrow y = \frac{3}{x}$$

$$3x + 2y = 3x + 2 \cdot \frac{3}{x} = 9$$

$$3x^2 + 6 = 9x$$

$$x^2 - 3x + 2 = 0$$

$$x_{1/2} = \frac{3}{2} \pm \sqrt{\frac{9}{4} - 2} = \frac{3}{2} \pm \frac{1}{2}$$

$$x_1 = 2 \Rightarrow y_1 = \frac{3}{2}$$

$$x_2 = 1 \Rightarrow y_2 = 3$$

$$10x + y = 10 \Rightarrow y = 10 - 10x$$

$$5x \cdot (15x + y) = 75$$

$$5x \cdot (15x + 10 - 10x) = 75$$

$$x(5x + 10) = 15$$

$$5x^2 + 10x - 15 = 0$$

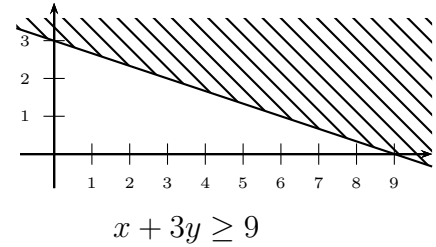
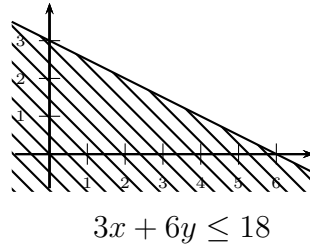
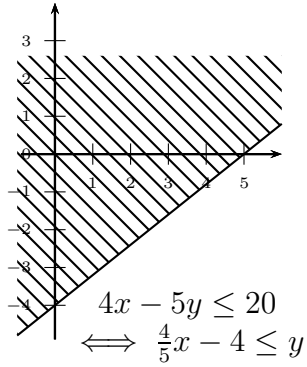
$$x^2 + 2x - 3 = 0$$

$$x_{1/2} = -1 \pm \sqrt{1 + 3} = -1 \pm 2$$

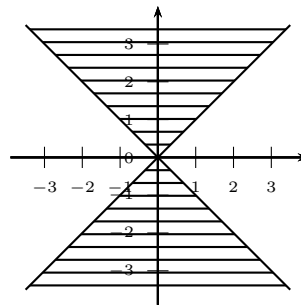
$$x_1 = -3 \Rightarrow y_1 = 40$$

$$x_2 = 1 \Rightarrow y_2 = 0$$

Lösung von Aufgabe 5:

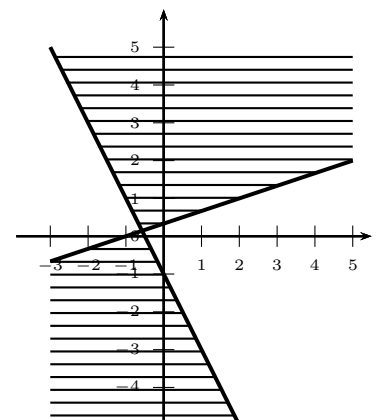
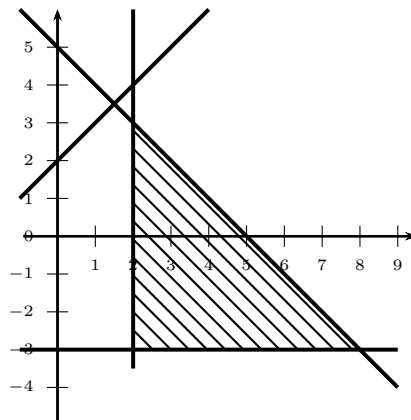
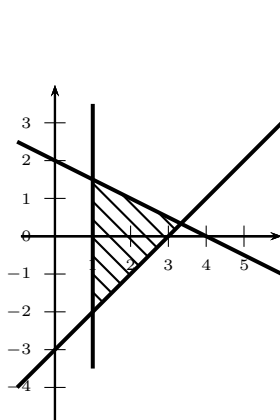


$$x^2 - y^2 \leq 0 \iff x^2 \leq y^2 \iff |x| \leq |y|$$



Lösung von Aufgabe 6:

$$\begin{array}{lll}
 x \geq 1 & x \geq 2 & (2x + y + 1)(3y - x - 1) \geq 0 \\
 x + 2y \leq 4 & -3 \leq y < 2 + x & \iff (2x + y + 1) \geq 0 \text{ und } (3y - x - 1) \geq 0 \\
 y + 3 \geq x & x + y \leq 5 & \text{oder: } (2x + y + 1) \leq 0 \text{ und } (3y - x - 1) \leq 0
 \end{array}$$



Lösung von Aufgabe 7:

a)

$$\begin{aligned}\frac{8x-5}{5} &\leq \frac{2x+5}{3} \\ 24x-15 &\leq 10x+25 \\ 14x &\leq 40 \\ x &\leq \frac{20}{7} \\ L &= \left\{ x \mid x \leq \frac{20}{7} \right\}\end{aligned}$$

b)

$$\frac{3}{2x-4} \leq 2$$

1. Fall: $x > 2$

$$\begin{aligned}\frac{3}{2x-4} &\leq 2 \\ \Leftrightarrow 3 &\leq 2(2x-4) \\ 3 &\leq 4x-8 \\ 11 &\leq 4x \\ \frac{11}{4} &\leq x \\ L_1 &= \left\{ x \mid \frac{11}{4} \leq x \right\}\end{aligned}$$

2. Fall: $x < 2$

$$\begin{aligned}\frac{3}{2x-4} &\leq 2 \\ \Leftrightarrow 3 &\geq 2(2x-4) \\ 3 &\geq 4x-8 \\ 11 &\geq 4x \\ \frac{11}{4} &\geq x \\ L_2 &= \{x \mid x < 2\}\end{aligned}$$

$$L = (-\infty; 2) \cup \left[\frac{11}{4}; \infty \right)$$

c)

$$\frac{2x-2}{2x+2} < 2$$

1. Fall: $x > -1$

$$\begin{aligned}\frac{2x-2}{2x+2} &< 2 \\ \Leftrightarrow 2x-2 &< 2(2x+2) \\ 2x-2 &< 4x+4 \\ -6 &< 2x \\ -3 &< x \\ L_1 &= \{x \mid x > -1\}\end{aligned}$$

2. Fall: $x < -1$

$$\begin{aligned}\frac{2x-2}{2x+2} &< 2 \\ \Leftrightarrow 2x-2 &> 2(2x+2) \\ 2x-2 &> 4x+4 \\ -6 &> 2x \\ -3 &> x \\ L_2 &= \{x \mid x < -3\}\end{aligned}$$

$$L = (-\infty; -3) \cup (-1; \infty)$$

d)

$$\frac{3}{4x-4} \leq \frac{2}{x-6}$$

1. Fall: $(x - 6)(4x - 4) > 0$,
d.h. $x > 1$ und $x > 6$ also $x > 6$
oder $x < 1$ und $x < 6$ also $x < 1$

$$\frac{3}{4x - 4} \leq \frac{2}{x - 6} \iff$$

$$3(x - 6) \leq 2(4x - 4)$$

$$3x - 18 \leq 8x - 8$$

$$-10 \leq 5x$$

$$-2 \leq x$$

$$L_1 = \{x \mid -2 \leq x < 1 \text{ oder } x > 6\}$$

$$= [-2; 1) \cup (6; \infty)$$

2. Fall: $(x - 6)(4x - 4) < 0$,
d.h. $1 < x < 6$

$$\frac{3}{4x - 4} \leq \frac{2}{x - 6} \iff$$

$$3(x - 6) \geq 2(4x - 4)$$

$$3x - 18 \geq 8x - 8$$

$$-10 \geq 5x$$

$$-2 \geq x$$

$$L_2 = \emptyset$$

$$L = \{x \mid -2 \leq x < 1 \text{ oder } x > 6\} = [-2; 1) \cup (6; \infty)$$

Lösung von Aufgabe 8:

$$(2x - 3)(3x - 2) < 0$$

ein Faktor positiv einer negativ: $2x - 3 < 0$ und $3x - 2 > 0 \iff x \in \left(\frac{2}{3}; \frac{3}{2}\right)$

oder andersrum: $2x - 3 > 0$ und $3x - 2 < 0$

$\iff x > \frac{3}{2}$ und gleichzeitig $x < \frac{2}{3}$ geht nicht

$$(x + 3)(7 - x) \leq 0 \iff x \leq -3 \text{ oder } x \geq 7 \Rightarrow L = (-\infty; -3] \cup [7; \infty)$$

$$x^2 + 4x + 3 \geq 0$$

$$x^2 + 4x + 3 = (x + 1)(x + 3) \geq 0$$

$$x + 1 \geq 0 \text{ und } x + 3 \geq 0 \Rightarrow x \geq -1$$

$$x + 1 \leq 0 \text{ und } x + 3 \leq 0 \Rightarrow x \leq -3$$

$$L = (-\infty; -3] \cup [-1; \infty)$$

$$2x^2 - 3x - 3 < 3(x - 1)$$

$$2x^2 - 3x - 3 < 3x - 3$$

$$2x^2 - 6x < 0$$

$$x^2 - 3x < 0$$

$$x(x - 3) < 0 \iff x - 3 < 0 \text{ und } x > 0$$

$$L = (0; 3)$$

Lösung von Aufgabe 9:

$$2 + x \geq 2x - 7 \Rightarrow 9 \geq x$$

$$5(x - 3) \geq 2(x - 3)$$

$$5x - 15 \geq 2x - 6$$

$$3x \geq 9$$

$$x \geq 3$$

$$\Rightarrow L = [3; 9]$$

$$x + 6 < 14 - 3x \iff 4x < 8 \iff x < 2$$
$$6 + 7x \geq 3x + 5 \iff 4x \geq -1 \iff x \geq -\frac{1}{4}$$

$$L = \left[-\frac{1}{4}; 2\right)$$

$$x^2 - 3x + 3 > 0$$

$$x^2 - 3x + 3 = 0 \iff x = \frac{3}{2} \pm \sqrt{\frac{9}{4} - 3} \quad \text{keine Nullstellen}$$

$$x^2 - 3x + 3 > 0 \quad \text{für alle } x \in \mathbb{R}$$

$$4(x - 1) < 2(x + 1)$$

$$4x - 4 < 2x + 2$$

$$2x < 6$$

$$x < 3$$

$$L = (-\infty; 3)$$